DELHI PUBLIC SCHOOL, DURGAPUR

QUESTION BANK FOR BLOCK TEST - 1 (2018-19)

CLASS-XII

SUB:MATHEMATICS

RELATIONS , FUNCTIONS AND BINARY OPERATION

1. Let A = $\{1,2,3,...9\}$ and R be the relation in AxA defined by (a,b)R(c,d), if a+d = b+c for (a,b), (c,d) in AxA . Prove that R is an equivalence relation and also obtain the equivalence relation $[\{2,5\}]$
2. Show that the relation R in the set A of real numbers defined as R = $\{(a,b): a \le b\}$ is reflexive and transitive but not symmetric.
3. Show that the relation R in the set Z of integers given by R = $\{(a,b): 2 divides (a-b)\}$ is an equivalence relation
4. Let R be the relation on R defined as (a , b) ∈ R iff 1+ ab> 0 ∀ a,b ∈ R. (a) Show that R is symmetric. (b) Show that R is reflexive. (c) Show that R is not transitive.
5. Show that the relation R on A ,A = { x| x ∈ Z , 0 ≤ x ≤ 12 } , R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.
6. Let A = {-1,0,1} and B = {0,1}. State whether the function f : A → B defined by f(x) = x^2 is bijective
7. Let * be a binary operation on the set Q of rational numbers defined as a * b = $\frac{ab}{5}$. Write the identity of *, if any.
8. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b). Find: (i) the identity element of A (ii) the invertible element of A.
9. Let A= N×N & * be a binary operation on A defined by (a , b) × (c , d) = (ac , bd) for all (a , b),(c , d) ∈ N×N (i) Find (2,3) * (4,1) (ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal (iii) Show that * is commutative & associative on A.
10. Show that the function f: $R \to R$, defined by f(x) = $\frac{x}{x^2+1}$,for all x∈R is neither one – one nor onto.
11. If the function f: $R \to R$,is given by f(x) = x^2 +2 and g: $R \to R$ is given by g(x) = $\frac{x}{x-1}; x \neq 1$, then find fog and gof and hence find fog(2) and gof (-3)
12. Consider $f: R_+ \to (-9,\infty)$ given by f(x) = $5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$ where R_+ is the set of all positive real numbers.
13. Consider the set S = $\{1,2,3,4\}$. Define a binary operation $*$ on S as follows : a$*$b=r, where r is the least non negative remainder , when ab is divided by 5 . Write the operation table for $*$

INVERSE TRIGONOMETRIC FUNCTION

1.If $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$, then find the value of x

2. Find the value of, $\tan^{-1}\left[2\cos\left\{2\sin^{-1}\frac{1}{2}\right\}\right]$

3. Write the function $\cot^{-1}\left(\sqrt{1+x^2}+x\right)$ in the simplest form.

4. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$

5. If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\pi$, prove that a+b+c =abc

6. If $\cos^{-1}a+\cos^{-1}b+\cos^{-1}c=\pi$, prove that $a^2+b^2+c^2+2abc=1$

7. Solve for x : $\tan^{-1}\frac{x+1}{x-1}+\tan\frac{x-1}{x}=\tan^{-1}(-7)$

8. Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

9. Solve for x : $\tan^{-1}x-\cot^{-1}x=\tan^{-1}\frac{1}{\sqrt{3}}$

10. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right)$

11. Prove that : $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

12. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$

13. Prove : $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\left(\frac{2b}{a}\right)$

14. Solve for x : $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}$

15. Find the value of the expression : $\sin(2\tan^{-1}\frac{1}{3})+\cos\left(\tan^{-1}2\sqrt{2}\right)$

DIFFERENTIATION

1. Determine the values of a,b, c for which the function $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, & x>0\end{cases}$ may be continuous at x = 0

2. Let $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$ Determine the value of a if possible so that the function is continuous at x = 0

3. Find the value of a for which the function f defined as $f(x)=\begin{cases}a\sin\frac{\pi}{2}(x+1), & x\le 0\\ \frac{\tan x-\sin x}{x^3}, & x>0\end{cases}$ is continuous at x = 0

4. Find the value of a and b such that the function f defined by $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, \text{if } x<4\\ a+b, if x=4\\ \frac{x-4}{|x-4|}+b, x>4\end{cases}$

is a continuous function at x = 4

5. Discuss the continuity of the function f(x) = $|x|+|x-1|$ at x = 1

6. Find the derivative of

(i) $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ (ii) $\tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0<x<\pi$ (iii) $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$

(iv) $y=\cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$ (v) $y=\cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ (vi) $y=\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\neq 0$ (vii) $y=x^{\cos x}+\sin x^{\tan x}$

7. If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$ 8. If f(x) = $\left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$

9. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for -1<x<1, show that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

10. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$

11. If x = a (cos t +t sin t) and y = a (sin t – t cost), then find $\frac{d^2y}{dx^2}$

12. If sin y = x sin(a+y), prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

13. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

14. If y = $e^{m\sin^{-1}x}$, prove that $(1-x^2)y_2-xy_1=m^2y$. 15. If y = $\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_2-xy_1+m^2y=0$

16. Differentiate $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, w.r.t. $\sqrt{1-x^2}$

17. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3(x^3-y^3)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

18. If $x = a\left\{\cos t + \log\left(\tan\frac{t}{2}\right)\right\}$; y = a sin t, show that $\frac{dy}{dx}=1$ at $t = \frac{\pi}{4}$

19. If x = sin t and y = sin pt, prove that $(1-x^2)y_2 - xy_1 + p^2y = 0$

20. If $y = Ae^{-kt}\cos(pt+c)$, then show that $y_2 + 2ky_1 + n^2y = 0$, where $n^2 = p^2 + k^2$

MATRICES AND DETERMINANT

1. Find x, y, z so that A = B, where $A = \begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix}, B = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$

2. Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

3. Prove the following by principle of mathematical induction : If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, for every positive integer n.

4. Use matrix multiplication to divide Rs 30, 000 in two parts such that the total amount interest at 9% on the first part and 11% on the second part amounts Rs 3060.

5. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence obtain A^3

6. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A^T A = I_2$

7. Express the matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew - symmetric matrix and verify your result

8. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|adjA|$

9. If A is an invertible matrix of order 3x3 such that $|A|=2$, then find adj(adjA)

10. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

11. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. Hence find A^{-1}

12. If $A=\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A^T)^{-1}$

13. Find the inverse of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by using elementary row transformations.

14. Solve the following system of equations by matrix method :

3x+4y +7z =14

2x-y +3z = 4

x+2y -3z = 0

15. $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations :

2x -3y +5z =11 , 3x+2y-4z = -5 , x+y -2z = -3

16.

A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of ₹ 6,000. Three times the award money for Hardwork added to that given for honesty amounts to ₹ 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

17. A total amount of Rs 7000 is deposited in three different saving bank accounts with annual interest rates 5% , 8% , and $8\frac{1}{2}$% respectively . The total annual interest from these three accounts is Rs 550 . Equal amounts have been deposited in the 5% and 8% saving accounts . Find the amount deposited in each of the three accounts , with the hel[p of matrices .

18. Evaluate without expanding :

(i) $\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$ (ii) $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$

19. Prove that

(i) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$ (ii) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(iii) $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ (iv) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

(v) $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$ (vi) $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

(vii) $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$ (viii) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

20. Solve : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

21. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) = abc+ab+bc+ca$

INDEFINITE INTEGRALS

Evaluate :

1. $\int \frac{1}{\sqrt{3x+4}-\sqrt{3x+1}}dx$ 2. $\int \frac{x^2+1}{(x+1)^2}dx$ 3. $\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$ 4. $\int \frac{\sin(x-a)}{\sin x}dx$

5. $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$ 6. $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$ 7. $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx$

8. $\int \frac{\tan x\sec^2 x}{(a+b\tan^2 x)^2}dx$ 9. $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx, \alpha \neq n\pi, n \in Z$ 10. $\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}dx$

11. $\int \sqrt{e^x-1}dx$ 12. $\int \frac{1}{x\sqrt{x^4-1}}dx$ 13. $\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx$ 14. $\int \sin^2 x\cos^5 x dx$

15. $\int \frac{1}{3x^2+13x-10}dx$ 16. $\int \frac{1}{x(x^5+1)}dx$ 17. $\int \frac{1}{\sqrt{(x-1)(x-2)}}dx$

18. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$ 19. $\int \frac{\sin 2x\cos 2x}{\sqrt{9-\cos^4 2x}}dx$ 20. $\int \sqrt{\sec x-1}dx$ 21. $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx$

22. $\int \frac{2x-3}{x^2+3x-18}dx$ 23. $\int \frac{x^2+1}{x^2-5x+6}dx$ 24. $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$ 25. $\int \sqrt{\frac{a-x}{a+x}}dx$

26. $\int \frac{1}{2-3\cos 2x}dx$ 27. $\int \frac{1}{\sqrt{3}\sin x+\cos x}dx$ 28. $\int \frac{3\cos x+2}{\sin x+2\cos x+3}dx$

29. $\int (\sin^{-1}x)^2 dx$ 30. $\int \cot^{-1}(1-x+x^2)dx$ 31. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$

32. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ 33. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$ 34. $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

35. $\int \left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$ 36. $\int e^x\frac{(1-x)^2}{(1+x^2)^2}dx$ 37. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$

38. $\int (3x-2)\sqrt{x^2+x+1}dx$ 39. $\int \frac{1}{\sin x-\sin 2x}dx$ 40. $\int \frac{1}{x+\sqrt{x^2-x+1}}dx$

41. $\int \frac{\sin x}{\sin 4x}dx$ 42. $\int \sqrt{\tan\theta}+\sqrt{\cot\theta}d\theta$ 43. $\int \frac{x^2}{x^4+1}dx$ 44. $\int \frac{1}{(x-1)\sqrt{x+2}}dx$

45. $\int \frac{1}{(x-1)\sqrt{x^2+1}}dx$ 46. $\int \frac{1}{(x^2+2x+2)\sqrt{x+1}}dx$ 47. $\int \frac{1}{(x^2-1)\sqrt{x^2+1}}dx$

48. $\int \frac{\sin x+cosx}{\sin^4 x+cos^4 x}dx$ 49. $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx$ 50. $\int \frac{\tan\theta+\tan^3\theta}{1+\tan^3\theta}d\theta$

DEFINITE INTEGRALS

Evaluate :

1. $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}dx$ 2. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ 3. $\int_0^1 |5x-3|dx$ 4. $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

5. $\int_0^{1.5} [x^2]dx$ 6. $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$ 7.

8. $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

9. $\int_0^{\pi} \frac{x}{1-\cos\alpha\sin x}dx$ 10. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3\sin^4 x dx$ 11. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ 12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\tan x}dx$

Evaluate the following definite integrals as limit of sums:

13. $\int_{-1}^{3} (x^2+3x)dx$ 14. $\int_1^3 (2x^2+5)dx$

AREA OF BOUNDED REGIONS

1. Using integration , find the area of the triangle ABC whose vertices have co ordinates A (2,5), B(4,7), C(6,2)
2. Sketch the graph y = $|x+1|$. Evaluate $\int_{-3}^{1} |x+1|dx$. What does this value represent on the graph?
3. Find the area bounded by the curve y = sinx between x = 0 and x = 2π
4. Find the area cut off the parabola 4y = $3x^2$ by the straight line 2y = 3x+12 (27sq units)
5. Indicate the region bounded by the curves x^2 =y , y = x+2 and x – axis and obtain the area enclosed by them . (5/6 sq units)
6. Find the area lying above the x – axis and included between the circle $x^2+y^2 = 8x$ and the parabola $y^2 = 4x$ $\frac{4}{3}(8+3\pi)$ sq units
7. Prove that the area bounded by the parabolas y^2 =-5x+6 and x^2 =y is 81/15 sq units
8. Find the area in the plane bounded by the curves y = x-1 and $(y-1)^2 = 4(x+1)$ 64/3 sq. units

9. Find the area of the region $\{(x,y): y^2 \le 4x, 4x^2+4y^2 \le 9\}$

$\left[\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right]$ sq units

10. Sketch the region common to the circle $x^2+y^2=16$ and the parabola $x^2=6y$. Also find the area of the region using integration. $\left[\frac{4\sqrt{3}}{3}+\frac{16\pi}{3}\right]$ sq units

APPLICATION OF DERIVATIVES
RATE MEASURES

1. The radius of a balloon is increasing at the rate of 10cm/sec. At what rate is the surface area of the balloon increasing when its radius is 15 cm ?

2. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which y-coordinate changes as fast as x – co ordinate.

3. A ladder 5 m long is leaning against the wall . The bottom of the ladder is pulled along the ground, away from the wall , at the rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall.

4. Water is passed into an inverted cone of base radius 5 cm and depth 10 cm at the rate of $\frac{3}{2}$ c.c/sec. Find the rate at which the level of the water is rising when depth is 4 cm .

5. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

6. A man 1.6 m tall walks at the rate of 0.5m/sec away from a lamp post , 8 mts high. Find the rate at which his shadow is increasing and the rate at which the tip of the shadow is moving away from the pole.

7. Water is dripping out from a conical funnel , at a uniform rate of 2 cm^3 /sec through a tiny hole at the vertex at the bottom , when the slant height of the water is 4 cm , find the rate of decrease of the slant height of the water , given that the vertical angle of the funnel is 120^0 .

8. Sand is pouring from a pipe at the rate of 12cm^3 /sec .The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base . How fast is the height of the sand – cone increasing , when the height is 4 cm ?

9. The volume of a cube is increasing at a constant rate .Prove that the increase in its surface area varies inversely as the length of the side.

10. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units /sec , then how fast is the slope of curve changing when x = 3 ?

11. Use differentials to approximate fourth root of 255

12. Find the approximate value of $\sqrt{0.0037}$

13. Find the approximate value of f(5.001), where $f(x) = x^3 - 7x^2 + 15$

14. Find the approximate change in the volume V of a cube of side x mts caused by increasing the side by 2%

15. If $y = x^4 - 10$ and if x changes from 2 to 1.97 , what is the approximate change in y ?

INCREASING AND DECREASING FUNCTION

1. Find the intervals in which the function f given by f(x) = sin x +cos x , $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing .

2. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x , throughout its domain.

3. Find the intervals in which the functions are increasing and decreasing

(i) $f(x) = 2x^3 - 9x^2 + 12x + 15$ (ii) $f(x) = (x-1)^3 (x-2)^2$ (iii) $\frac{4\sin x - 2x - x\cos x}{2+\cos x}, 0 \le x \le 2\pi$

(iv) $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$ (v) $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ (vi) $f(x) = \frac{4x^2+1}{x}$

(vii) $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$

4. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on the interval $(0, \frac{\pi}{2})$

5. Prove that $f(\theta) = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

6. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on (-1,1)

7. Find the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on $[1,2]$.

Also find the greatest value of 'a' for which f(x) is decreasing on $[1,2]$

8. Show that $f(x) = e^{\frac{1}{x}}, x \neq 0$ is a decreasing function for all $x \neq 0$

9. Separate $\left[0, \frac{\pi}{2}\right]$ into sub - intervals in which f(x) = sin 3x is increasing or decreasing.

10.Separate $\left[0, \frac{\pi}{2}\right]$ into sub - intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

TANGENTS AND NORMALS

1. Prove that the tangents to the curves $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) are at right angles .

2. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

3. At what point on the curves $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the y – axis ?

4. Find the points on the curve $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at which the tangents are parallel to the (i) x – axis

(ii) y – axis

5. Find the co ordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

6. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.

7. Find the points on the curve $4x^2 + 9y^2 = 1$, where the tangents are perpendicular to the line $2y + x = 0$

8. Find the equation of the tangent line to the curve $x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$

9. Find the equations of the tangents and normal at the point 't' on the curve $x = a\sin^3 t$, $y = b\cos^3 t$

10. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 2$ which is parallel to the line

$4x - 2y + 3 = 0$

11. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1,2)

12. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the points (1,2)

13. Prove that all normals to the curve $x = a\cos t + at\sin t$, $y = a\sin t - at\cos t$ are at a distance a from the origin

14. Find the equations of all lines of slope zero and that are tangent to the curve y = $y = \frac{1}{x^2 - 2x + 3}$

15. Prove that $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ for all $n \in N$, at the point (a,b)

16. Find the condition for the curves $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$ and $xy = c^2$ to intersect orthogonally .

MAXIMA AND MINIMA

1. Show that of all the rectangles inscribed in a given circle, the square has the maximum area

2. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given , show that the area of

the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

3. Prove that the area of right – angled triangle of given hypotenuse is maximum when the triangle is isosceles.

4. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

5. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

6. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What would be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

7. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm

8. Show that the height of the closed cylinder of given surface and maximum volume, is equal to the diameter of its base.

DIFFERENTIAL EQUATION

1. Determine the order and degree of the :

(i) $\frac{d^3y}{dx^3}+\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}+4y=\sin x$ (ii) $\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)=0$

2. Form the differential equation corresponding to $y^2 = a(b-x)(b+x)$ by eliminating parameters a and b

3. Find the differential equation of all circles in the first quadrant which touch the coordinate axes.

4. Verify that y = $e^{m\cos^{-1}x}$ satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$

5. Solve the following differential equations:

(i) $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ (ii) $(x+1)\frac{dy}{dx}=2e^{-y}-1, y(0)=0$ (iii) $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

(iv) $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$ (v) $(x^2-y^2)dx+2xy\,dy=0$, given that y = 1 when x = 1

(vi) $(3xy+y^2)dx+(x^2+xy)dy=0$ (vii) $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0, y(1)=\frac{\pi}{2}$

(viii) $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x, x>0$ (ix) $\frac{dy}{dx}=-\frac{x+y\cos x}{1+\sin x}$ (x) $(x-\sin y)dy+(\tan y)dx=0$, $y(0)=0$

(xi) $\sqrt{1-y^2}dx=(\sin^{-1}y-x)dy, y(0)=0$

LINEAR PROGRAMMING

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C.Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg. to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.
2. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.
3. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

SYLLABUS FOR BLOCK TEST – 1

Relations and Functions, Inverse Trigonometric Functions, Matrices, Determinants, Continuity and Differentiability

Differentiation , Applications of Derivatives, Integrals, Applications of the Integrals, Differential Equations, Linear Programming